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A comparison of the non-standard effects of effective theories with linearly realized and strongly interacting Higgs sectors*)

Robert F. Szalapski

Theory Group, KEK, 1-1 Oho, Tsukuba, Ibaraki 305, Japan
E-mail address: robs@theory.kek.jp

Abstract

Effective theories provide a powerful tool for testing the Standard Model and for searching for the effects of new physics in a model-independent manner. In general one assumes that the effects of new physics characterized by a high-energy scale may be observed at low-energies only through quantum corrections. These quantum corrections may be described by effective operators which are constructed from the fields of the low-energy theory; the coefficients of these operators may be measured via experiment, but a theoretical determination is only possible once the complete theory is known. Because the existence or non-existence of a light physical Higgs boson has not yet been established the inclusion or exclusion of the Higgs field in the construction of the effective Lagrangian is open to debate. For this reason we discuss both the linearly realized effective Lagrangian, which includes a light physical Higgs boson, and the chiral Lagrangian, where the Higgs boson has been removed by a formal integration. In each scenario the contributions to oblique parameters and to W-pair-production amplitudes are discussed. Where possible relationships between the two effective Lagrangians are given.

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§1. The linear representation and energy-dimension-six operators

If the scale of new physics, Λ, is large compared to the vacuum expectation value (vev) of the the Higgs field, $v = 246.22$GeV, then the effective Lagrangian may be expressed as the SM Lagrangian plus terms with energy dimension greater than four suppressed by inverse powers of Λ, *i.e.*

$$\mathcal{L}_{eff} = \mathcal{L}_{\rm SM} + \sum_{n\geq 5}\sum_{i} \frac{f_i^{(n)}\,\mathcal{O}_i^{(n)}}{\Lambda^{n-4}} \ . \qquad (1\cdot 1)$$

The energy dimension of each operator is denoted by n, and the index i sums over all operators of the given energy dimension. The coefficients $f_i^{(n)}$ are free parameters.

In this section we assume that the low-energy theory, *i.e.*the SM, contains a light physical scalar Higgs particle which is the remnant of a complex Higgs-doublet field. We assume that the couplings of the new physics to fermions are suppressed, and we only consider operators which conserve CP. Upon restricting the analysis to operators not exceeding energy-dimension six we find that twelve operators form a basis set; all are dimension-six and separately conserve C and P [1]. They are summarized in Table I. For convenience when defining the normalizations of the individual operators we use the 'hatted' field strength tensors defined according to

$$\Big[D_\mu, D_\nu\Big] = \hat{W}_{\mu\nu} + \hat{B}_{\mu\nu} = igT^a W^a_{\mu\nu} + ig' Y B_{\mu\nu} \ . \qquad (1\cdot 2)$$

Combining the twelve operators of Table I with Eqn. (1·1) completes the construction of the effective Lagrangian in the linear representation.

Also in Table I we indicate those vertices to which each operator contributes with an 'X' in the appropriate box. First, observe that four of the operators, $\mathcal{O}_{DW}$, $\mathcal{O}_{DB}$, $\mathcal{O}_{BW}$ and $\mathcal{O}_{\Phi,1}$, contribute to gauge-boson two-point-functions at the tree level. For this reason their respective coefficients, f_{DW}, f_{DB}, f_{BW} and $f_{\Phi,1}$, are strongly constrained by LEP/SLC and low-energy data [2], and these constraints will be improved by data at higher energy lepton colliders. (This will be discussed in greater detail in Sec. 3.) These four operators will contribute to the process $e^+e^- \to W^+W^-$ through corrections to the charge form-factors, $\bar{e}^2(q^2)$, $\bar{s}^2(q^2)$, $\bar{g}_Z^2(q^2)$ and $\bar{g}^2(q^2)$, and through the W-boson wave-function-renormalization factor, $Z_W^{1/2}$.

The operators $\mathcal{O}_{DW}$ and $\mathcal{O}_{BW}$ also make a direct contribution to $WW\gamma$ and WWZ vertices. Three additional operators contribute as well. They are $\mathcal{O}_{WWW}$, $\mathcal{O}_W$ and $\mathcal{O}_B$; their respective coefficients are f_{WWW}, f_W and f_B. The non-zero direct contributions may be summarized by

$$\Delta g^\gamma_{1,{\rm direct}}(q^2) = 2\hat{g}^2 \frac{q^2 + 2\hat{m}_W^2}{\Lambda^2} f_{DW} \ , \qquad (1\cdot 3a)$$

$$\Delta g^Z_{1,\text{direct}}(q^2) = 2\hat{g}^2\frac{q^2+2\hat{m}_W^2}{\Lambda^2}f_{DW} + \frac{1}{2}\frac{\hat{m}_Z^2}{\Lambda^2}f_W\ , \tag{1·3b}$$

$$\Delta\kappa_{\gamma,\text{direct}}(q^2) = 2\hat{g}^2\frac{q^2+2\hat{m}_W^2}{\Lambda^2}f_{DW} + \frac{1}{2}\frac{\hat{m}_W^2}{\Lambda^2}\Big(f_W - 2f_{BW} + f_B\Big)\ , \tag{1·3c}$$

$$\Delta\kappa_{Z,\text{direct}}(q^2) = 2\hat{g}^2\frac{q^2+2\hat{m}_W^2}{\Lambda^2}f_{DW} + \frac{1}{2}\frac{\hat{m}_Z^2}{\Lambda^2}\Big(\hat{c}^2 f_W + 2\hat{s}^2 f_{BW} - \hat{s}^2 f_B\Big)\ , \tag{1·3d}$$

$$\begin{aligned}\Delta\lambda_{\gamma,\text{direct}}(q^2) &= \Delta\lambda_{Z,\text{direct}}(q^2)\\ &= \frac{3}{2}\hat{g}^2\Big(-4f_{DW} + f_{WWW}\Big)\frac{\hat{m}_W^2}{\Lambda^2}\ .\end{aligned} \tag{1·3e}$$

Electromagnetic gauge invariance requires that Δg_1^γ should vanish for on-shell photons, in apparent contradiction with (1·3a); the apparent contradiction will be resolved when all effects are included.

Naively one would expect contributions to (1·3) and to the gauge-boson two-point-functions from $\mathcal{O}_{WW}$ and $\mathcal{O}_{BB}$. However, their contributions may be completely absorbed by a redefinition of SM fields and gauge couplings, leading to a null contribution. For this reason an 'O' is used for these operators in Table I. Additionally $\mathcal{O}_{\Phi,4}$ contributes to the W- and Z-mass terms, while $\mathcal{O}_{\Phi,1}$ contributes to the Z-mass term only. Hence $\mathcal{O}_{\Phi,1}$ violates the custodial symmetry, and the T parameter is explicitly dependent upon $f_{\Phi,1}$. On the other hand, the contributions from $\mathcal{O}_{\Phi,4}$ exactly cancel in the calculation of T, hence it does not contribute.

§2. The non-linear realization and operators of the electroweak chiral Lagrangian.

It is possible that there is no physical Higgs boson, and the mechanism of spontaneous symmetry breaking is not part of the SM, but a part of its extension. In such a scenario the full Lagrangian may be written as

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_i \mathcal{L}_i + \cdots\ . \tag{2·1}$$

In contrast to the linearly realized Lagrangian (1·1), the leading correction terms in the chiral Lagrangian are not suppressed by inverse powers of some high scale.

In order to proceed it is convenient to introduce some additional notation. The charge-conjugate Higgs field may be written $\Phi^c = i\tau^2\Phi^*$. Then

$$U \equiv \frac{\sqrt{2}}{v}\Big(\Phi^c, \Phi\Big) \longrightarrow \mathbf{1}\ , \tag{2·2a}$$

$\mathcal{O}_i^{(6)}$	WW	ZZ	AZ	AA	WWZ	WWA	WWWW	WWZZ	WWZA	WWAA	ZZZZ
$\mathcal{O}_{DW} = \mathrm{Tr}\left(\left[D_\mu, \hat{W}_{\nu\rho}\right]\left[D^\mu, \hat{W}^{\nu\rho}\right]\right)$	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	
$\mathcal{O}_{DB} = -\frac{g'^2}{2}\left(\partial_\mu B_{\nu\rho}\right)\left(\partial^\mu B^{\nu\rho}\right)$		**X**	**X**	**X**							
$\mathcal{O}_{BW} = \Phi^\dagger \hat{B}_{\mu\nu}\hat{W}^{\mu\nu}\Phi$		**X**	**X**	**X**	**X**	**X**					
$\mathcal{O}_{\Phi,1} = \left[\left(D_\mu\Phi\right)^\dagger \Phi\right]\left[\Phi^\dagger\left(D^\mu\Phi\right)\right]$		**X**									
$\mathcal{O}_{WWW} = \mathrm{Tr}\left(\hat{W}_{\mu\nu}\hat{W}^{\nu\rho}\hat{W}_\rho{}^\mu\right)$					**X**	**X**	**X**	**X**	**X**	**X**	
$\mathcal{O}_{WW} = \Phi^\dagger \hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\Phi$	**O**	**O**	**O**	**O**	**O**	**O**	**O**	**O**	**O**	**O**	
$\mathcal{O}_{BB} = \Phi^\dagger \hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\Phi$	**O**	**O**	**O**	**O**	**O**	**O**	**O**	**O**	**O**	**O**	
$\mathcal{O}_{W} = \left(D_\mu\Phi\right)^\dagger \hat{W}^{\mu\nu}\left(D_\nu\Phi\right)$					**X**	**X**	**X**	**X**	**X**		
$\mathcal{O}_{B} = \left(D_\mu\Phi\right)^\dagger \hat{B}^{\mu\nu}\left(D_\nu\Phi\right)$					**X**	**X**					
$\mathcal{O}_{\Phi,2} = \frac{1}{2}\partial_\mu\left(\Phi^\dagger\Phi\right)\partial^\mu\left(\Phi^\dagger\Phi\right)$											
$\mathcal{O}_{\Phi,3} = \frac{1}{3}\left(\Phi^\dagger\Phi\right)^3$											
$\mathcal{O}_{\Phi,4} = \left(\Phi^\dagger\Phi\right)\left[\left(D_\mu\Phi\right)^\dagger\left(D^\mu\Phi\right)\right]$	**O**	**O**									

Table I. Energy-dimension-six operators in the linear representation of the Higgs mechanism. The contribution of an operator to a particular vertex is denoted by an **'X'** . In some cases an operator naively contributes to a vertex, yet that contribution does not lead to observable effects. In such cases the **'X'** is replaced by an **'O'** .

$$D_\mu U = \partial_\mu U + igT^a W_\mu^a U - ig' U T^3 B_\mu \longrightarrow igT^a W_\mu^a - ig' T^3 B_\mu \ , \tag{2·2b}$$

$$T \equiv 2UT^3U^\dagger \longrightarrow 2T^3 \ , \tag{2·2c}$$

$$V_\mu \equiv (D_\mu U)U^\dagger \longrightarrow D_\mu U \ . \tag{2·2d}$$

The arrows indicate the unitary-gauge expression. In the notation of Appelquist and Wu [3] we present a list of chiral operators through energy-dimension four which conserve CP. There

are twelve such operators given by

$$\mathcal{L}_1' = \frac{\beta_1 v^2}{4}\left[\mathrm{Tr}\,(TV_\mu)\right]^2\,, \tag{2·3a}$$

$$\mathcal{L}_1 = \frac{\alpha_1 g g'}{2} B_{\mu\nu}\mathrm{Tr}\left(TW^{\mu\nu}\right)\,, \tag{2·3b}$$

$$\mathcal{L}_2 = \frac{i\alpha_2 g'}{2} B_{\mu\nu}\mathrm{Tr}\left(T[V^\mu, V^\nu]\right)\,, \tag{2·3c}$$

$$\mathcal{L}_3 = i\alpha_3 g\,\mathrm{Tr}\left(W_{\mu\nu}[V^\mu, V^\nu]\right)\,, \tag{2·3d}$$

$$\mathcal{L}_4 = \alpha_4\left[\mathrm{Tr}\,(V_\mu V_\nu)\right]^2\,, \tag{2·3e}$$

$$\mathcal{L}_5 = \alpha_5\left[\mathrm{Tr}\,(V_\mu V^\mu)\right]^2\,, \tag{2·3f}$$

$$\mathcal{L}_6 = \alpha_6 \mathrm{Tr}\left(V_\mu V_\nu\right)\mathrm{Tr}\left(TV^\mu\right)\mathrm{Tr}\left(TV^\nu\right)\,, \tag{2·3g}$$

$$\mathcal{L}_7 = \alpha_7 \mathrm{Tr}\left(V_\mu V^\mu\right)\mathrm{Tr}\left(TV_\nu\right)\mathrm{Tr}\left(TV^\nu\right)\,, \tag{2·3h}$$

$$\mathcal{L}_8 = \frac{\alpha_8 g^2}{4}\left[\mathrm{Tr}\,(TW_{\mu\nu})\right]^2\,, \tag{2·3i}$$

$$\mathcal{L}_9 = \frac{i\alpha_9 g}{2}\mathrm{Tr}\left(TW_{\mu\nu}\right)\mathrm{Tr}\left(T[V^\mu, V^\nu]\right)\,, \tag{2·3j}$$

$$\mathcal{L}_{10} = \frac{\alpha_{10}}{2}\left[\mathrm{Tr}\,(TV_\mu)\mathrm{Tr}\,(TV_\nu)\right]^2\,, \tag{2·3k}$$

$$\mathcal{L}_{11} = \alpha_{11}\, g\, \epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\left(TV_\mu\right)\mathrm{Tr}\left(V_\nu W_{\rho\sigma}\right)\,. \tag{2·3l}$$

The last operator, $\mathcal{L}_{11}$, violates parity, P. The operators are summarised in Table II, which employs the same format as Table I. Additionally Table II pairs each chiral operator with its linearly realized counterpart, four of which appear in Sec. 1. The remainder, which occur at the energy-dimension-eight, -ten and -twelve level may be found elsewhere [4)].

Three of the chiral operators, $\mathcal{L}_1'$, $\mathcal{L}_1$ and $\mathcal{L}_8$, contribute to gauge-boson two-point-functions. Like $\mathcal{O}_{\Phi,1}$, $\mathcal{L}_1'$ contributes only to the Z-mass term but not to the W-mass term and leads to a violation of the custodial symmetry. Through contributions to the charge form-factors $\bar{e}^2(q^2)$, $\bar{s}^2(q^2)$, $\bar{g}_Z^2(q^2)$ and $\bar{g}^2(q^2)$ these three operators will contribute to the process $e^+e^- \to W^+W^-$. None of the operators contributes to the WW two-point-function, hence, in contrast to the linear realization, no contribution will be made via the W-boson wave-function-renormalization factor, and the t-channel terms are not modified.

$\mathcal{L}_{\text{chiral}}$	$\mathcal{O}^{(n)}_{\text{linear}}$	WW	ZZ	AZ	AA	WWZ	WWA	WWWW	WWZZ	WWZA	WWAA	ZZZZ
$\mathcal{L}'_1$	$-\frac{4\beta_1}{v^2}\mathcal{O}_{\Phi,1}$		**X**									
$\mathcal{L}_1$	$\frac{4\alpha_1}{v^2}\mathcal{O}_{BW}$		**X**	**X**	**X**	**X**	**X**					
$\mathcal{L}_2$	$\frac{8\alpha_2}{v^2}\mathcal{O}_{B}$					**X**	**X**					
$\mathcal{L}_3$	$\frac{8\alpha_3}{v^2}\mathcal{O}_{W}$					**X**	**X**	**X**	**X**	**X**		
$\mathcal{L}_4$	$\frac{4\alpha_4}{v^4}\mathcal{O}^{(8)}_{4}$							**X**	**X**			**X**
$\mathcal{L}_5$	$\frac{16\alpha_5}{v^4}\mathcal{O}^{(8)}_{5}$							**X**	**X**			**X**
$\mathcal{L}_6$	$-\frac{64\alpha_6}{v^6}\mathcal{O}^{(10)}_{6}$								**X**			**X**
$\mathcal{L}_7$	$-\frac{64\alpha_7}{v^6}\mathcal{O}^{(10)}_{7}$								**X**			**X**
$\mathcal{L}_8$	$-\frac{4\alpha_8}{v^4}\mathcal{O}^{(8)}_{8}$		**X**	**X**	**X**	**X**	**X**	**X**				
$\mathcal{L}_9$	$-\frac{16\alpha_9}{v^4}\mathcal{O}^{(8)}_{9}$					**X**	**X**	**X**				
$\mathcal{L}_{10}$	$\frac{128\,\alpha_{10}}{v^8}\mathcal{O}^{(12)}_{10}$											**X**
$\mathcal{L}_{11}$	$\frac{8\alpha_{11}}{v^4}\mathcal{O}^{(8)}_{11}$					**X**				**X**		

Table II. Column one lists energy-dimension-four operators in the non-linear representation. The linear-representation counterparts appear in the second column. For the definitions of the operators $\mathcal{O}^{(n)}_i$ the reader is referred to the text. An **'X'** is used to indicate the the contribution of an individual operator to a particular vertex.

In total six of the operators, $\mathcal{L}_1$, $\mathcal{L}_2$, $\mathcal{L}_3$, $\mathcal{L}_8$, $\mathcal{L}_9$ and $\mathcal{L}_{11}$, contribute directly to three-gauge-boson vertices. The non-zero corrections are

$$\Delta g^{Z,\text{nl}}_{1,\text{direct}} = \hat{g}^2_Z \alpha_3 \; , \tag{2·4a}$$

$$\Delta\kappa^{\text{nl}}_{\gamma,\text{direct}} = \hat{g}^2\Big(-\alpha_1 + \alpha_2 + \alpha_3 - \alpha_8 + \alpha_9 \Big) \; , \tag{2·4b}$$

$$\Delta\kappa^{\text{nl}}_{Z,\text{direct}} = \hat{g}^2_Z \hat{s}^2 \Big(\alpha_1 - \alpha_2 \Big) + \hat{g}^2 \Big(\alpha_3 - \alpha_8 + \alpha_9 \Big) \; , \tag{2·4c}$$

$$\Delta g^{Z,\text{nl}}_{5,\text{direct}} = \hat{g}^2_Z \alpha_{11} \; . \tag{2·4d}$$

Notice that here, unlike Eqn. (1·3a), $\Delta g_{1,\rm direct}^{\gamma,\rm nl}$ is trivially zero. Furthermore notice that $\Delta\lambda_{\gamma,\rm direct}^{\rm nl}$ and $\Delta\lambda_{Z,\rm direct}^{\rm nl}$ are identically zero; these couplings are associated with energy-dimension-six operators, which are higher order corrections in the present scheme.

§3. Four-fermion processes

First we present the contributions of the linearly realized operators of Table I to the oblique parameters. Due to the presence of $(q^2)^2$ terms in the gauge-boson two-point-functions the original derivative-based definitions of S, T and U [5] are not convenient. A more appropriate scheme for present purposes is where the derivative is replaced by a finite difference [6] of the form

$$\overline{\Pi}^{AB}_{T,V}(q^2) = \frac{\overline{\Pi}^{AB}_{T}(q^2) - \overline{\Pi}^{AB}_{T}(m_V^2)}{q^2 - m_V^2}\ . \tag{3·1}$$

Then

$$\Delta S \equiv 16\pi\,\mathcal{R}e\left[\Delta\overline{\Pi}^{3Q}_{T,\gamma}(m_Z^2) - \Delta\overline{\Pi}^{33}_{T,Z}(0)\right] = -4\pi\frac{v^2}{\Lambda^2}f_{BW}\ , \tag{3·2a}$$

$$\Delta T \equiv \frac{4\sqrt{2}G_F}{\alpha}\,\mathcal{R}e\left[\Delta\overline{\Pi}^{33}_{T}(0) - \Delta\overline{\Pi}^{11}_{T}(0)\right] = -\frac{1}{2\alpha}\frac{v^2}{\Lambda^2}f_{\Phi,1}\ , \tag{3·2b}$$

$$\Delta U \equiv\ 16\pi\,\mathcal{R}e\left[\Delta\overline{\Pi}^{33}_{T,Z}(0) - \Delta\overline{\Pi}^{11}_{T,W}(0)\right]\ = 32\pi\frac{m_Z^2 - m_W^2}{\Lambda^2}f_{DW}\ , \tag{3·2c}$$

where $S = S_{\rm SM} + \Delta S$, $T = T_{\rm SM} + \Delta T$ and $U = U_{\rm SM} + \Delta U$. Because the $f_{\Phi,1}$ and $f_{\Phi,4}$ contributions to the two-point functions are independent of q^2 they may contribute only to T. The $f_{\Phi,4}$ contributions exactly cancel, as was 'predicted' during the discussion of Table I. The $(q^2)^2$ terms in the two-point functions also lead to a non-standard running of the SM charge form-factors. The combination of S, T and U with the non-standard running leads to the convenient expressions

$$\Delta\overline{\alpha}(q^2) = -8\pi\hat{\alpha}^2\frac{q^2}{\Lambda^2}\Big(f_{DW} + f_{DB}\Big)\ , \tag{3·3a}$$

$$\Delta\overline{g}_Z^2(q^2) = -2\hat{g}_Z^4\frac{q^2}{\Lambda^2}\Big(\hat{c}^4 f_{DW} + \hat{s}^4 f_{DB}\Big) - \frac{1}{2}\hat{g}_Z^2\frac{v^2}{\Lambda^2}f_{\Phi,1}\ , \tag{3·3b}$$

$$\begin{aligned}\Delta\overline{s}^2(q^2) = \frac{-\hat{s}^2\hat{c}^2}{\hat{c}^2 - \hat{s}^2}&\left[8\pi\hat{\alpha}\frac{m_Z^2}{\Lambda^2}\Big(f_{DW} + f_{DB}\Big) + \frac{m_Z^2}{\Lambda^2}f_{BW} - \frac{1}{2}\frac{v^2}{\Lambda^2}f_{\Phi,1}\right]\\ &+ 8\pi\hat{\alpha}\frac{q^2 - m_Z^2}{\Lambda^2}\Big(\hat{c}^2 f_{DW} - \hat{s}^2 f_{DB}\Big)\ ,\end{aligned} \tag{3·3c}$$

$$\Delta\overline{g}_W^2(q^2) = -8\pi\hat{\alpha}\hat{g}^2\frac{m_Z^2}{\Lambda^2}f_{DB} - \hat{g}^2\frac{\Delta\overline{s}^2(m_Z^2)}{\hat{s}^2} - \frac{1}{4}\hat{g}^4\frac{v^2}{\Lambda^2}f_{BW} - 2\hat{g}^4\frac{q^2}{\Lambda^2}f_{DW}\ . \quad (3{\cdot}3\mathrm{d})$$

The 'hatted' couplings satisfy the tree-level relationships $\hat{e} \equiv \hat{g}\hat{s} \equiv \hat{g}_Z\hat{s}\hat{c}$ and $\hat{e}^2 \equiv 4\pi\hat{\alpha}$.

The calculations may be repeated for the non-linear representation. In this case the dependence on q^2 is at most linear, hence there is no non-standard running of the charge form-factors. The oblique parameters are given by

$$\Delta S = -16\pi\alpha_1\ , \quad (3{\cdot}4\mathrm{a})$$
$$\Delta T = \frac{2}{\alpha}\beta_1\ , \quad (3{\cdot}4\mathrm{b})$$
$$\Delta U = -16\pi\alpha_8\ , \quad (3{\cdot}4\mathrm{c})$$

which agrees with [3]. The contributions to the charge form-factors may be written

$$\Delta\overline{\alpha}(q^2) = 0\ , \quad (3{\cdot}5\mathrm{a})$$
$$\Delta\overline{g}_Z^2(q^2) = 2\hat{g}_Z^2\beta_1\ , \quad (3{\cdot}5\mathrm{b})$$
$$\Delta\overline{s}^2(q^2) = -\frac{\hat{c}^2\hat{s}^2}{\hat{c}^2-\hat{s}^2}\left(2\beta_1 + \hat{g}_Z^2\alpha_1\right)\ , \quad (3{\cdot}5\mathrm{c})$$
$$\Delta\overline{g}_W^2(q^2) = -\hat{g}^2\frac{\Delta\overline{s}^2(m_Z^2)}{\hat{s}^2} - \hat{g}^4\left(\alpha_1 + \alpha_8\right)\ . \quad (3{\cdot}5\mathrm{d})$$

The low-energy data severely constrain those parameters of the chiral Lagrangian which contribute to S, T and U; these constraints may be improved via improved measurements at low energies. The corresponding parameters of the linearly realized Lagrangian are less stringently constrained at present. However, because some of their contributions to observables are enhanced by higher powers of q^2 these constraints may be expected to improve significantly at LEP2 and beyond [2].

§4. Corrections to $e^+e^- \to W^+W^-$

The most general amplitude for $e^+e^- \to W^+W^-$, depicted in Figure 1 may be written

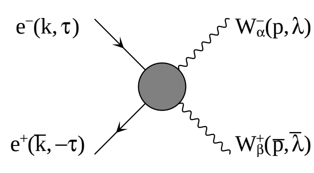

Fig. 1. The process $ee \to W^+W^-$ with momentum and helicity assignments. All momenta are defined flowing to the right. In the massless-electron limit $\overline{\tau} = -\tau$.

$$\mathcal{M}(k,\bar{k},\tau;p,\bar{p},\lambda,\bar{\lambda}) = \sum_{i=1}^{9} F_{i,\tau}(s,t)\, j_\mu(k,\bar{k},\tau) T_i^{\mu\alpha\beta}\epsilon_\alpha^*(p,\lambda)\epsilon_\beta^*(\bar{p},\bar{\lambda})\ , \quad (4{\cdot}1)$$

where all dynamical information is contained in the scalar form-factors $F_{i,\tau}(s,t)$. The other factors in Eqn. (4·1) are of a purely kinematical nature; $\epsilon^*_\alpha(p,\lambda)$ and $\epsilon^*_\beta(\bar{p},\bar{\lambda})$ are the polarization vectors for the W^- and W^+ bosons respectively, and $j_\mu(k,\bar{k},\tau)$ is the fermion current for massless electrons. It is necessary to include nine tensors, $T_i^{\mu\alpha\beta}$, to be completely general. One possible choice is

$$T_1^{\mu\alpha\beta} = P^\mu g^{\alpha\beta} \ , \tag{4·2a}$$

$$T_2^{\mu\alpha\beta} = \frac{-1}{m_W^2} P^\mu q^\alpha q^\beta \ , \tag{4·2b}$$

$$T_3^{\mu\alpha\beta} = q^\alpha g^{\mu\beta} - q^\beta g^{\alpha\mu} \ , \tag{4·2c}$$

$$T_4^{\mu\alpha\beta} = i\Big(q^\alpha g^{\mu\beta} + q^\beta g^{\alpha\mu} \Big) \ , \tag{4·2d}$$

$$T_5^{\mu\alpha\beta} = i\epsilon^{\mu\alpha\beta\rho} P_\rho \ , \tag{4·2e}$$

$$T_6^{\mu\alpha\beta} = -\epsilon^{\mu\alpha\beta\rho} q_\rho \ , \tag{4·2f}$$

$$T_7^{\mu\alpha\beta} = \frac{-1}{m_W^2} P^\mu \epsilon^{\alpha\beta\rho\sigma} q_\rho P_\sigma \ , \tag{4·2g}$$

$$T_8^{\mu\alpha\beta} = K^\beta g^{\alpha\mu} + K^\alpha g^{\mu\beta} \ , \tag{4·2h}$$

$$T_9^{\mu\alpha\beta} = \frac{i}{m_W^2} \Big(K^\alpha \epsilon^{\beta\mu\rho\sigma} + K^\beta \epsilon^{\alpha\mu\rho\sigma} \Big) q_\rho P_\sigma \ , \tag{4·2i}$$

where $P = p - \bar{p}$ and $K = k - \bar{k}$. This completely determines the kinematics, hence we focus on the form factors $F_{i,\tau}(s,t)$.

We include only the tree-level contributions of the effective Lagrangians from Sec. 1 and Sec. 2. If we choose the renormalization conditions $\hat{s}^2 = \bar{s}^2(q^2)$ and $\hat{e}^2 = \bar{e}^2(q^2)$ the expressions for the $F_{i,\tau}(s,t)$ are somewhat simplified. We write

$$F_{i,\tau}(s,t) = \frac{1}{s} Q \hat{e}^2 f^\gamma_{i,\tau} + \frac{1}{s - m_Z^2 + is\frac{\Gamma_Z}{m_Z}\Theta(s)} \Big(I_3 - \hat{s}^2 Q \Big) \hat{c}^2 \hat{g}_Z^2 f^Z_{i,\tau} + \frac{1}{2t} I_3 \hat{g}^2 f^t_{i,\tau} \ . \tag{4·3}$$

The amplitudes for $e^+e^- \to W^+W^-$ are completely determined once the lower-case form-factors, f_i^X, are specified. The results for the SM, which are particularly simple, may be found in Table III. These values guarantee the gauge cancellations which prevent unitarity violations in amplitudes which involve one or more longitudinally polarized W boson.

We may calculate the form-factors with effective-Lagrangian contributions. In general we expect results that will spoil the gauge cancellations of the SM. The non-zero form-factors in the linear representation, for the photonic contributions, may be written

$$f^\gamma_{1,\pm} = 1 + \frac{\Delta\bar{e}^2(s)}{\hat{e}^2} + \hat{g}^2 \frac{s}{\Lambda^2} \Big(- f_{DW} + \frac{3}{4} f_{WWW} \Big) \ , \tag{4·4a}$$

	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$	$i=8$	$i=9$
$f^{\gamma}_{i,\pm}$	1	0	2	0	0	0	0	0	0
$f^{Z}_{i,\pm}$	1	0	2	0	0	0	0	0	0
$f^{t}_{i,-}$	1	0	2	0	1	0	0	1	0

Table III. Explicit values for the form factors of the SM at the tree level.

$$f^{\gamma}_{2,\pm} = \frac{3}{2}\hat{g}^2\frac{m_W^2}{\Lambda^2}\Bigg(-4f_{DW} + f_{WWW}\Bigg) , \tag{4·4b}$$

$$f^{\gamma}_{3,\pm} = 2 + 2\frac{\Delta\bar{e}^2(s)}{\hat{e}^2} + 2\hat{g}^2\frac{2s-3m_W^2}{\Lambda^2}f_{DW} + \frac{3}{2}\hat{g}^2\frac{m_W^2}{\Lambda^2}f_{WWW} + \frac{1}{2}\frac{m_W^2}{\Lambda^2}\Bigg(f_W - 2f_{BW} + f_B\Bigg) , \tag{4·4c}$$

where $\Delta\bar{e}^2(q^2)$ is obtained from (3·3a). For the Z-boson

$$f^{Z}_{1,\pm} = 1 + \frac{\Delta\bar{g}_Z^2(s)}{\hat{g}_Z^2} + \hat{g}^2\frac{s}{\Lambda^2}\Bigg(-f_{DW} + \frac{3}{4}f_{WWW}\Bigg) + \frac{1}{2}\frac{m_Z^2}{\Lambda^2}f_W , \tag{4·5a}$$

$$f^{Z}_{2,\pm} = \frac{3}{2}\hat{g}^2\frac{m_W^2}{\Lambda^2}\Bigg(-4f_{DW} + f_{WWW}\Bigg) , \tag{4·5b}$$

$$f^{Z}_{3,\pm} = 2 + 2\frac{\Delta\bar{g}_Z^2(s)}{\hat{g}_Z^2} + 2\hat{g}^2\frac{2s-3m_W^2}{\Lambda^2}f_{DW} + \frac{3}{2}\hat{g}^2\frac{m_W^2}{\Lambda^2}f_{WWW} + \frac{1}{2}\frac{m_Z^2}{\Lambda^2}\Bigg[\Bigg(1+\hat{c}^2\Bigg)f_W + 2\hat{s}^2 f_{BW} - \hat{s}^2 f_B\Bigg] , \tag{4·5c}$$

with $\Delta\bar{g}_Z^2(q^2)$ given by (3·3b). In the t-channel

$$f^{t}_{1,-}\frac{1}{2} = f^{t\,(\mathrm{eff})}_{3,-} = f^{t}_{5,-} = f^{t\,(\mathrm{eff})}_{8,-} = \frac{\bar{g}_W^2(m_W^2)}{\hat{g}^2} , \tag{4·6}$$

with $\bar{g}_W^2(m_W^2)$ given by Eqn. (3·3d).

The calculation may be repeated for the chiral Lagrangian of Sec. 2. For the $f^{\gamma}_{i,\tau}$ we find

$$f^{\gamma}_{1,\pm} = 1 , \tag{4·7a}$$

$$f^{\gamma}_{3,\pm} = 2 + \hat{g}^2(-\alpha_1 + \alpha_2 + \alpha_3 - \alpha_8 + \alpha_9) . \tag{4·7b}$$

while for $f^{Z}_{i,\tau}$

$$f^{Z}_{1,\pm} = 1 + 2\beta_1 + \hat{g}_Z^2\alpha_3 , \tag{4·8a}$$

$$f^{Z}_{3,\pm} = 2 + 4\beta_1 + \hat{g}_Z^2\hat{s}^2(\alpha_1 - \alpha_2) + \hat{g}_Z^2(1+\hat{c}^2)\alpha_3 + \hat{g}^2(-\alpha_8 + \alpha_9) , \tag{4·8b}$$

$$f^{Z}_{5,\pm} = \hat{g}_Z^2\alpha_{11} . \tag{4·8c}$$

For the t-channel form-factors,

$$f^{t}_{1,-} = \frac{1}{2} f^{t\,(\mathrm{eff})}_{3,-} = f^{t}_{5,-} = f^{t\,(\mathrm{eff})}_{8,-} = 1 + \frac{2\hat{g}^2}{\hat{c}^2 - \hat{s}^2} \left(\hat{c}^2 \beta_1 + \hat{e}^2 \alpha_1 \right) - \hat{g}^4 \alpha_8 \; , \qquad (4{\cdot}9)$$

The above contributions in general violate the gauge cancellations of the SM. There is not sufficient space for a detailed discussion. Notice, however, that the results are consistent with electromagnetic gauge invariance. We may calculate $g_1^\gamma = f_1^\gamma - (q^2/2m_W^2) f_2^\gamma$, and we anticipate that the result should be $g_1^\gamma = 1$ for on-shell photons. For the linearly realized scenario $\Delta g_1^\gamma = g_1^\gamma - 1$ is proportional to q^2, while for the chiral Lagrangian $\Delta g_1^\gamma = 0$; both satisfy the gauge-invariance condition.

§5. Conclusions

I have outlined the contributions of the effective theories to processes with four external fermions. In this sector extensive numerical studies have been completed which include complete SM radiative corrections [1), 2)].

I also presented complete amplitudes for $e^+e^- \to W^+W^-$. While in either realization of the Higgs sector seven operators contribute to these amplitudes, in the linear (non-linear) realization four (three) are already constrained via four-fermion processes. Hence the number of free parameters is very manageable, and this may prove to be an extremely useful tool for the analysis of LEP2 and linear collider data.

It is expected that LEP2 will not be sensitive to the SM radiative corrections to $e^+e^- \to W^+W^-$, hence the present analysis, which treats the non-standard effects as large compared to SM radiative corrections, is of immediate interest. However, eventually it will be necessary to include complete SM corrections. The analysis presented is sufficiently general to include the complete corrections; Eqn. 4·1 is completely general, though the expressions for the $F_{i,\tau}(s,t)$ become more complicated than Eqn. 4·3.

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